

Linear Algebra: Assessed Questions 1

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1 Assessed questions

A total of 10 marks is available.

Exercise 1.1. For each geometrical transformation below, write down the 3-by-3 matrix which encodes it.

1 marks

- (a.i) X : rotation by 180 degrees around the x -axis.
- (a.ii) Y : rotation by 180 degrees around the y -axis.
- (a.iii) Z : rotation by 180 degrees around the z -axis.

True or false? Justify your answer.

- (b.i) X followed by Y gives Z . 1 marks
- (b.ii) $XY = YX$. 1 marks
- (b.iii) $XYZ = -I$. 1 marks

Exercise 1.2. The *trace* of a matrix is defined to be the sum of its diagonal entries. Write down the trace of a matrix A_{ij} using index notation and prove that $\text{Tr}(AB) = \text{Tr}(BA)$.

2 marks

Exercise 1.3. Consider the matrix:

$$M = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix}$$

True, false or nonsense? Justify your answers.

- (a) There exists a vector $v = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$ such that $Mv = 0$. 1 marks
- (b) There exists a vector $w = \begin{pmatrix} x \\ y \end{pmatrix}$ such that $Mw = 0$. 1 marks
- (c) For any vector $w = \begin{pmatrix} x \\ y \end{pmatrix}$ there exists a vector $v = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$ such that $Mv = w$. 2 marks