Linear Algebra: Workshop Questions 1

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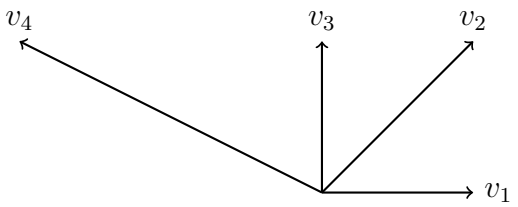
1 Questions

Exercise 1.1. Which of these vectors is $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$?



Exercise 1.2. Compute the matrix product $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$.

Exercise 1.3. Take the matrix $M = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ and the vector $w = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Apply M to w. Then apply M again. Then apply M again. Continue until you spot a pattern. Can you express the pattern as a formula? Can you prove that this pattern is going to continue? *Hint: You may write F_n for the nth term in a certain famous sequence of numbers.*

Exercise 1.4. Does there exist a matrix A such that $A^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$? *Hint: Think geometrically about what A^2 does.*

Exercise 1.5. True or false:

(a) If M is a matrix and v is a particular vector such that $Mv = v$ then M is the identity matrix.

(b) If $M^2 = M$ then either $M = I$ or $M = 0$.

Challenge 1.6. Given an integer a, define $M(a) := \begin{pmatrix} a & 1 \\ 1 & 0 \end{pmatrix}$. Show that if

$$\begin{pmatrix} p \\ q \end{pmatrix} = M(a_1)M(a_2) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

then

$$\frac{p}{q} = a_1 + \frac{1}{a_2}.$$

What would the formula look like if you had more factors $M(a_k)$?