# Linear Algebra: Workshop Questions 2

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## 1 Questions

**Exercise 1.1.** Let $u = \begin{pmatrix} 1 \\ 0 \\ -2 \\ 2 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$.

(a) What is the angle between $u$ and $v$?

(b) Write down a vector $w \in \mathbb{R}^4$ which is orthogonal to both $u$ and $v$.

**Exercise 1.2.** Let

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}, \qquad B = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & 0 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

(a) Which of these two matrices is orthogonal?

(b) The orthogonal matrix from part (a) represents a rotation. What are the axis and angle of this rotation?

**Exercise 1.3.** True or false? If false, give a counterexample.

(a) Any 1-by-$n$ matrix is in echelon form.

(b) Let$A$ be a matrix in reduced echelon form. If $A$ has no free variables then the equation $Av = b$ always has a solution.

**Exercise 1.4.** Write this system of equations as a matrix equation. Put the corresponding augmented matrix into reduced echelon form and hence determine whether there is a solution or not.

$$\begin{aligned} 2x + 4y - 5z &= -7 \\ -x - 2y + z &= 1 \\ x + 2y &= 1 \end{aligned}$$

**Exercise 1.5.** (a) A system of simultaneous equations is called *overdetermined* if there are more equations than variables. It is true that overdetermined systems never have solutions?

(b) A system of simultaneous equations is called *underdetermined* if there are more variables than equations. It is true that underdetermined systems always have solutions?