# Linear Algebra: Assessed Questions 4

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## 1 Assessed questions

A total of 10 marks is available.

**Exercise 1.1.** Find the eigenvalues and eigenvectors of the matrix **5 marks**

$$M = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}.$$

What are the dimensions of the eigenspaces? **1 marks**

**Exercise 1.2.** Let $\mathcal{M}$ denote the set of 4-by-4 matrices whose characteristic polynomial is $(\lambda - 1)(\lambda - 2)(\lambda - 3)^2$.

(a) Find an $A \in \mathcal{M}$ such that all of the eigenspaces of $A$ are 1-dimensional. **1 marks**

(b) Find a $B \in \mathcal{M}$ such that at least one eigenspace of $B$ is 2-dimensional. **1 marks**

(c) Is it true that $C \in \mathcal{M}$ implies $C$ is invertible? **1 marks**

(d) Is it true that, for any $D \in \mathcal{M}$, no positive power of $D$ equals the identity? **1 marks**