Linear Algebra: Workshop Questions 4

Jonny Evans

1 Questions

Exercise 1.1. True or false? In each case, give a proof or a counterexample.

(a) If v is both a λ-eigenvector of A and a μ-eigenvector of B then v is a $(\lambda+\mu)$-eigenvector of $A+B$.

(b) If A is an n-by-n matrix and m is a number then $\det(mA) = m^n \det(A)$.

(c) For any matrix A and any eigenvalue λ of A, any two λ-eigenvectors v, w are related by rescaling, i.e. $w = cv$ for some number c.

Exercise 1.2. Find the eigenvalues and eigenvectors of the matrix $M = \begin{pmatrix} -4 & -25 \\ 1 & 6 \end{pmatrix}$.

Exercise 1.3. Let c be a number. What is the characteristic polynomial of the matrix $M = \begin{pmatrix} c & 1 & 0 \\ 1 & c & 1 \\ 0 & 1 & c \end{pmatrix}$? What are its eigenvectors and eigenvalues?

Exercise 1.4. By expanding about a row or column, evaluate the determinant

$$\det \begin{pmatrix} t & 0 & 1 \\ 3 & 1 & 2 \\ 1 & 1 & 0 \end{pmatrix}$$

and state the values of t for which this matrix fails to be invertible.

Exercise 1.5. The *Pell numbers* are the sequence P_n of integers starting $0, 1, 2, 5, 12, 29, \ldots$ satisfying the recurrence relation

$$P_{n+2} = 2P_{n+1} + P_n.$$

Write down a matrix M such that

$$M \begin{pmatrix} P_{n+1} \\ P_n \end{pmatrix} = \begin{pmatrix} P_{n+2} \\ P_{n+1} \end{pmatrix}.$$

Find the eigenvectors and eigenvalues of M.