

Linear Algebra: End of module questions

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1 End of module questions

A total of 10 marks is available.

Exercise 1.1. Let $c > 0$ be a positive real number. Your answers will depend on c .

Consider the matrix $M = \begin{pmatrix} c & 2 \\ 2 & 1 \end{pmatrix}$.

- (a) Find the characteristic polynomial of M . **2 marks**
- (b) Find the eigenvalues of M . **2 marks**
- (c) For which values of c are both eigenvalues positive? **2 marks**
- (d) If $c = 5$, find the eigenvectors of M . **2 marks**
- (e) Sketch the ellipse $cx^2 + 4xy + y^2 = 1$ for $c = 5$. **1 marks**
- (f) By thinking about the eigenvalues as $c \rightarrow \infty$, can you describe (roughly) what happens to the shape of this ellipse as c increases? **1 marks**