Topology and Groups

Week 9, Monday

1 Preparation

- 8.02 (Covering transformations),
- 8.03 (Normal covers).

2 Discussion

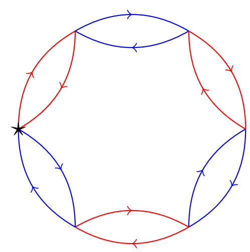
1. Why is there a covering isomorphism $F\colon (Y_1, p_1) \to (Y_2, p_2)$ with $F(y_1) = y_2$ if and only if $(p_1)_*\pi_1(Y_1, y_1) = (p_2)_*\pi_1(Y_2, y_2)$ as subgroups of $\pi_1(X, x)$?
2. (PCQ) In the proof that covering transformations are covering maps, why is $q \circ (p_2)|_W$ a local inverse for F?
3. (PCQ) Because I was using the lifting criterion, I should have added an assumption about my spaces in the existence and uniqueness theorem for covering transformations. What should I have said?

3 Classwork

3.1 Covering transformations and normal covers

1. Suppose that X is a space with $\pi_1(X, x) = S_3$ (permutation group on three objects) and that we have covering spaces $p_1\colon Y_1 \to X$, $p_2\colon Y_2 \to X$ with $(p_1)_*\pi_1(Y_1, y_1) = \{1, (12)\}$ and $(p_2)_*\pi_1(Y_2, y_2) = \{1, (13)\}$. Is there a covering transformation from Y_1 to Y_2?
2. Let $p_m\colon S^1 \to S^1$ be the covering map $p_m(z) = z^m$. How many covering transformations are there $(S^1, p_m) \to (S^1, p_n)$?

3. Consider the covering space in the figure below. Why is it normal? Give a normal generating set for the subgroup of $\langle a, b\rangle$ associated to this covering space.



3.2 Simply-connected covering spaces

1. Suppose X admits a simply-connnected covering space. How many such covering spaces does it admit up to covering isomorphism?
2. Suppose that X admits a simply-connected covering space $p\colon Y \to X$ and that $p'\colon Y' \to X$ is another covering space. Show that Y is a covering space of Y.
3. What are the universal covers of the following spaces?